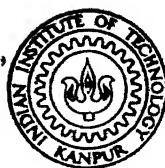


# OPTIMIZATION OF THE FIN GEOMETRY AND INTER-FIN DISTANCE FOR FORCED CONVECTION LAMINAR FLOW

By  
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DEPARTMENT OF MECHANICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY KANPUR

JULY 1973

# OPTIMIZATION OF THE FIN GEOMETRY AND INTER-FIN DISTANCE FOR FORCED CONVECTION LAMINAR FLOW

A Thesis Submitted  
In Partial Fulfilment of the Requirements  
for the Degree of  
MASTER OF TECHNOLOGY

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AWTAR NARAIN MATHUR

to the  
DEPARTMENT OF MECHANICAL ENGINEERING  
INDIAN INSTITUTE OF TECHNOLOGY KANPUR  
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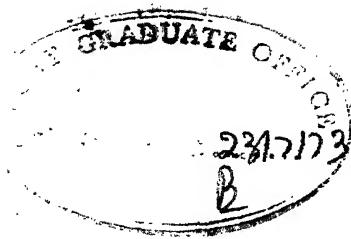
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TO MY PARENTS



### CERTIFICATE

Certified that this work on 'OPTIMIZATION OF THE FIN GEOMETRY AND INTER-FIN DISTANCE FOR FORCED CONVECTION LAMINAR FLOW' by Awtar Narain Mathur has been carried out under my supervision and that this has not been submitted elsewhere for a degree.

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This thesis has been approved  
for the award of the degree of  
Master of Engineering (Tech.)

In accordance with the  
regulations of the  
Institute of Technology Kanpur  
Dated. 3.8.73 24

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## LIST OF SYMBOLS

$B_i$	Biot number
$b$	Thickness of fin
$CV$	Characteristic Value
$d_1, d_o$	Tube and fin diameter respectively
$h, h_c, h_r$	Heat transfer coefficient, convective, radiative
$I_0, I_1, K_0, K_1$	Bessel functions
$k, k_a$	Thermal conductivity of metal and air respectively
$L$	Characteristic length
$n$	Number of fins
$Pr$	Prandtl number
$Q$	Heat transfer rate
$r_1, r_o$	Radius of tube and fin respectively
$r_k$	Penalty Parameter
$Re$	Reynolds number
$s$	Inter-fin distance
$U, U_\infty$	Velocity of flow
$\theta_1$	Temperature difference between tube and surrounding
$\theta$	Temperature difference between fin and surrounding
$\nu$	Kinematic viscosity of air
$\sim$	Notation for representing a vector

SYNOPSIS  
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JULY, 1973  
OPTIMIZATION OF THE FIN GEOMETRY AND INTER-FIN  
DISTANCE FOR FORCED CONVECTION LAMINAR FLOW

In this dissertation the optimum fin geometry and inter-fin distance have been obtained using recently developed optimization techniques. The volume of fins is sought to be minimum subjected to the following constraints :

1. Increased heat transfer in comparison to that of bare tube.
2. The boundary layer interference.
3. Biot number for optimum utility of the fin surfaces.
4. Number of fins.
5. Geometrical constraints.

Available algorithms of the constrained minimization have been applied to the heat transfer problem.

The electrical network analogy is used to find the temperature drop in the fin. The heat transfer rate has been calculated by using the electrical analogy method.

For the optimum fin, with minimum volume, heat transfer rate is increased to 40-50 times that of the bare tube.

The temperature drop and the heat transfer rate computed by the network analogy method are found to be in excellent agreement with those of numerically computed values.

## CHAPTER 1

### INTRODUCTION

#### 1.1 INTRODUCTION

Any heat transfer surface which has metallic projections meant to increase the surface area available for heat transfer, is called a finned surface. The metallic projections are referred to as fins. A tube provided with fins is called a finned tube.

Different geometries, such as spines of triangular, circular, rectangular (with constant or variable thickness), circular wedge shapes etc. can be used for fins. Annular and constant thickness rectangular fins are simple to manufacture. Fins may be transverse or longitudinal.

Fins may be welded to the primary heat transferring surface or integrally cast. Sometimes metal ribbons are peened to tubes. Circular fins may be shrunk fitted along the tubes.

An improved grooved and peened fin tube known as keyfin has been developed by the Hunt Heat Exchanger Company [1]. It gives a positive bond between the extended and primary surfaces. Soft brazing is most common to ensure contact.

It is argued that the substitution of porous metal for fins quadruples the efficiency of heat exchangers and evaporators at a cost which is 2 to 3 times that of ordinary tubings, while saving 75% copper [2]. This is open for further exploration.

In the present work, the problem of optimization of the fin geometry and inter-fin distance for aluminium has been solved for minimum volume and increased heat transfer. Forced flow under steady state is assumed. The various constraints on the problem are :

1. Increased heat transfer in comparison to that of a bare tube
2. Flow conditions
3. Biot number based on the transverse fin dimension
4. Boundary layer interference
5. Number of fins.

Nusselt relationship has been used for calculating the heat transfer coefficient. The radiation effects have not been accounted for, as the temperature difference between the fin and the environment is small.

The temperature distribution in the fin has been obtained by electrical analogy methods and the heat transfer rate has been verified for the best fin.

### 1.2 LITERATURE REVIEW

Carrier and Anderson [3] have considered the case of straight and annular fins of constant thickness and calculated the heat transfer, comparing them with bare tubes and thus computed efficiencies.

Daughtry [4] has obtained a relationship for short fins and long fins with and without insulated ends. He has compared

the results and plotted curves to compare them with bare tubes.

Murray and Cambridge [5] have shown that the addition of a fin changes the temperature distribution around the tube. This change is negligible in slowly moving air.

Gardner [6] obtained equations for heat transferred from a finned tube as well as fin efficiency, in the form of Bessel functions. He has considered annular fins and spines of constant and varying cross-sections in steady state, assuming constant surface temperature and one dimensional conduction. The first assumption is seriously questionable in the case of heat exchangers though it is valid for evaporators and condensers.

Wilkins [7] has shown that for the given rate of heat transfer at constant base temperature, the optimum fin profile has a very sharp point end and a triangular profile uses about 63.5% less material than the other profiles like straight or parabolic.

Edward and Chaddock [8] were the first to account for radiation from the finned surface. Radiation can take place between tube and fin surfaces, between two adjacent fins, fin and enclosure, and tube and enclosure. The first two contribute negligible amount and may be neglected, but the last two contribute significant amount to heat transfer. The experiment was for natural convection and heat transfer coefficient was assumed to be the function of temperature.

Th. E. Schimdt [9] has determined the profile of the fin requiring the least material. He considered the contribution of

each surface of fin and tube separately to heat transfer, by varying the fin size. In order to compare different shapes he defines fin effectiveness as the ratio of the actual heat dissipated by the fin surface to that which would be dissipated if the fin surface were held at the temperature of the base. He proves that the tip of the fin should approach the temperature of the surrounding fluid.

It is of primary interest to recognise the conditions for which the finned surface has advantages over the unfinned surface. Daughtry [4] compared the heat transfer through the base of three fins : (a) short fin, (b) the long fin with insulated end, and (c) long fin with heat loss from the end. He concludes that the use of too short a fin will violate the assumption of one dimensional conduction. For heat transfer between air and gases, fins are advantageous if the characteristic value (reciprocal of Biot modulus) defined by Eq. (1-1) is sufficiently high.

$$CV = 1/Bi = 2k/(h b), \quad (1-1)$$

where  $k$  - the thermal conductivity of fin material,  
 $h$  - the heat transfer coefficient between fin and surrounding fluid,  
 $b$  - the thickness of the fin.

If this value is small ( $\ll 0.02$ ) the use of fins results in no increase in heat transfer over that for a bare tube.

Hence the selection of thinner fins of high conductive metal is always preferred.

Hutcheon and Spalding [10] analysed the problem of using an analogue computer for free convection. They assumed that heat transfer from the finned surface was proportional to 5/4th power of difference of temperature. A remarkable feature of their work is that they accounted for heat transfer from the fin tip. Their results, compared with a case where heat transfer is proportional to difference of temperature, have been seen to be more accurate.

Recently, Dent [11] has solved the case of an annular fin for natural convection by the equivalent electrical network method using an analogue computer. Dent has accounted for heat transfer by radiation also, as suggested by Edward and Chaddock [8] by using the same expression for shape factor. He has thus obtained fairly good results by developing an iterative scheme for use with the computer.

Danilova and Dyundin [12] have indicated that heat transfer coefficient for finned tube is higher than that for a smooth tube. The heat transfer coefficient from a finned tube has been shown to depend on factors such as the surrounding fluid, its temperature, heat flux, fin geometry and inter-fin distance. The reduction of inter-fin distance to some extent increases the heat transfer coefficient.

Hutcheon and Spalding [10] have shown that there is a very little difference in the calculated rate of heat transfer, even if the heat transfer coefficient is considered as constant instead of variable in free convection, under steady state conditions. McAdams [14] has said that heat transfer coefficient varies from point to point for certain types of fins, but if average conductivities and average coefficients are used for calculating the temperature gradient and heat transfer rate, the results are in excellent agreement with experimental results.

Katz et al [15] put forth an expression for heat transfer coefficient for a finned tube as a function of fin outer diameter and spacing and later established it experimentally. The expression suggested is found to be in excellent agreement with the available values for natural flow and condensing conditions. The results have been verified for various velocities of air flow.

### 1.3 MOTIVATION FOR PRESENT WORK

Optimization of fin geometry and inter-fin distance was not considered until 1949. Fin geometry was optimized for maximum heat transfer rate maintaining the fin radius constant. Edward and Chaddock [8] optimized the inter-fin distance experimentally using annular fins of constant thickness for free-convection. Their results were verified by Dent [11].

## CHAPTER II

### HEAT TRANSFERRED FROM FINNED TUBE

#### 2.1 INTRODUCTION

The use of finned tubes is advisable only if the characteristic Biot number based on transverse fin dimension is very small. Biot number should always be less than unity, hence the thermal conductivities of fin materials should be high. Biot number based on transverse fin length :

$$h b/k \ll 1.0 \quad (2-1)$$

The material chosen for fin is aluminium which has a thermal conductivity of 175 kcal/m-hr-°C. The heat transfer coefficient for air is between  $10-100 \text{ kcal/m}^2\text{-hr-}^{\circ}\text{C}$  and for water as medium  $500-5000 \text{ kcal/m}^2\text{-hr-}^{\circ}\text{C}$ . Hence the use of aluminium fins in the air is justified but for water it is seldom recommended. For a fin of thickness 2.5 mm the Biot number is,

$$Bi = (10) (.0025)/175 \quad (2-2)$$

$$\ll 1.0$$

As the dominating mode of heat transfer from the tube is convection, fins add to the heat transfer rate, when the area of heat transfer increases. If the height of fins on a tube is comparatively small with respect to tube diameter, the formula to calculate the heat transfer rate for a plane wall can also be

## CHAPTER II

## HEAT TRANSFERRED FROM FINNED TUBE

2.1 INTRODUCTION

The use of finned tubes is advisable only if the characteristic Biot number based on transverse fin dimension is very small. Biot number should always be less than unity, hence the thermal conductivities of all materials should be high. Biot number based on transverse fin length

$$h b/k \ll 10 \quad (2-1)$$

The material chosen for fin is aluminium which has a thermal conductivity of 175 kcal/m-hr-°C. The heat transfer coefficient for air is between 10-100 kcal/m<sup>2</sup>-hr-°C and for water as medium 500-5000 kcal/m<sup>2</sup>-hr-°C. Hence the use of aluminium fins in the air is justified but for water it is seldom recommended. For a fin of thickness 2.5 mm the Biot number is,

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As the dominating mode of heat transfer from the tube is convection, fins add to the heat transfer rate, when the area of heat transfer increases. If the height of fins on a tube is comparatively small with respect to tube diameter, the formula to calculate the heat transfer rate for a plane wall can also be

used for a finned tube. For pin type rectangular fins, heat transfer can be calculated by using the formula for a straight rod with appropriate boundary conditions. Using these formulae it can be established that the use of fins over unfinned surfaces is advantageous if the characteristic value is greater than 5. Then, an optimum fin geometry can be predicted [13].

When fins are advantageous, the heat transfer can be increased by placing them as near one another as practicable. The heat transfer coefficient decreases when the boundary layers, which occur on the surfaces of the two adjacent fins, mutually influence each other. The distance between two fins must not be appreciably smaller than twice the boundary layer thickness. Comparison between theory and experiment shows good agreement in computed and measured heat transfer coefficients, until the distance between fins is less than about 1/3rd the diameter of tube. But, if the distance between the fins is decreased further, the difference becomes greater.

Air flowing along a flat plate of 30 cm length with a velocity of 15 m/sec creates a boundary layer, approximately 2.5 mm thick. A lower velocity ( $< 1$  m/sec) results in larger boundary layer, about 12.5 mm thick.

## 2.2 ASSUMPTIONS IN CALCULATING HEAT TRANSFER FROM FINNED TUBE

The following assumptions have been used to calculate heat transfer from finned tubes :

1. Steady state conditions exist.
2. Air flowing across fins has uniform temperature.
3. The temperature of tube surface is uniform.
4. As the thickness of the fin is small, as compared to fin diameter and inter-fin distance, the temperature gradient, normal to fin surface, is negligible.
5. Ends of fins do not transfer significant amounts of energy as compared with other parts of the fin i.e. the ends are insulated.
6. Heat transfer coefficient is the same all over the fin and the tube surfaces.
7. There is no heat source present in the fin.
8. Fin material is homogeneous and isotropic and its conductivity is constant.

A few of the assumptions listed above are seriously questionable. The heat transfer coefficient does vary from point to point on the fin, but the use of average conductivity and coefficient in the theoretical calculation for heat transfer and temperature gradient gives good agreement with experimental results [6].

The temperature of the tube is measured along its periphery and then averaged. It justifies assumption 3.

The tip area of the fin is negligibly small in comparison to the total heat transfer area, and hence it adds less than 2% to the total heat transfer [6]. Thus its effect can safely be as stated in assumption, neglected and ends can be taken to be insulated as assumed in 5. This will be discussed further while discussing the results.

### 2.3 HEAT TRANSFER FROM FIN

Because of relatively high thermal conductivity of fin metal it has been found possible to consider the temperature uniform at any cross section. This means that the conduction is one dimensional [9].

Considering Fig. 2-1, and assuming steady state conditions, an energy balance on a differential ring of area  $dA$  yields

$$Q_{\text{cond.}} = Q_{\text{conv.}} + Q_{\text{rad.}} \quad (2-3)$$

Net conduction into element

$$Q_{\text{cond.}} = A(x) k \left( \frac{d\theta}{dx} \right) = 2\pi b k \left[ r \frac{d^2\theta}{dr^2} + \frac{d\theta}{dr} \right] dr \quad (2-4)$$

Total convective transfer from the surface of element is

$$Q_{\text{conv.}} = 4\pi r h \theta dr \quad (2-5)$$

In accounting for the radiative exchanges with the differential ring, three separate transfers must be considered :

- (1) that between the tube and element,
- (2) that between the enclosure and element,
- and (3) that between the element on one fin and on an adjacent fin.

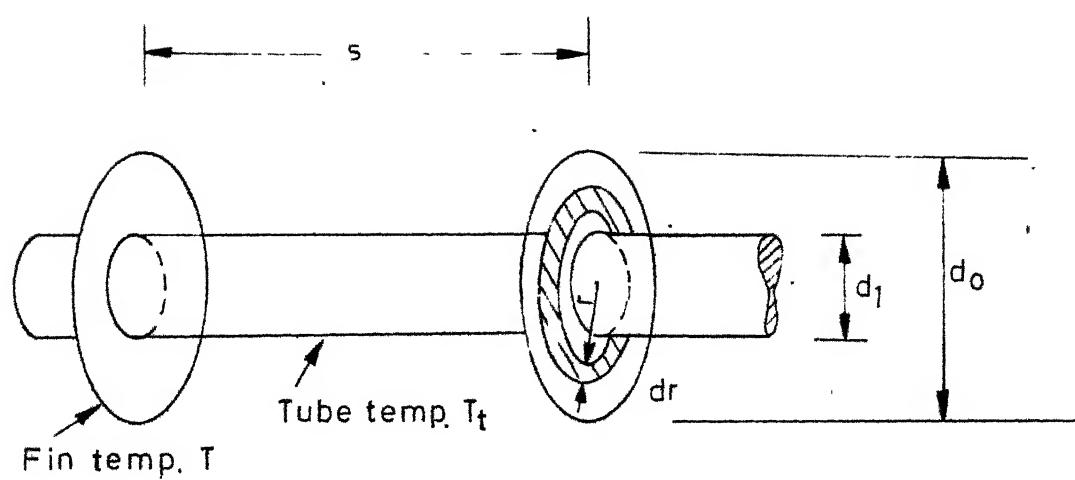


Fig. 2-1 Finned tube with differential area  $dA$

For a small temperature difference between the tube and the surroundings, the total radiation energy amounts to a small quantity as it is proportional to the difference of fourth power of absolute temperature; hence Eq. (2-3) becomes :

$$\frac{d^2\theta}{dr^2} + \frac{1}{r} \frac{d\theta}{dr} - \frac{2h}{kb} \theta = 0 \quad (2-6)$$

At the base of the fin the temperature is the same as that of the tube which leads, along with assumption 5, to the boundary condition :

$$\begin{aligned} r &= r_1 & \theta &= \theta_1, \\ \text{and } r &= r_0 & \frac{d\theta}{dr} &= 0. \end{aligned} \quad (2-7)$$

Equation (2-6) along with the boundary conditions of Eq. (2-7) has been solved in terms of modified Bessel functions [18] and verified numerically :

$$\theta = \theta_1 \frac{K_1(r_0 \sqrt{\beta}) I_0(r_1 \sqrt{\beta}) + I_1(r_0 \sqrt{\beta}) K_0(r_1 \sqrt{\beta})}{K_1(r_0 \sqrt{\beta}) I_0(r_1 \sqrt{\beta}) + I_1(r_0 \sqrt{\beta}) K_0(r_1 \sqrt{\beta})} \quad (2-8)$$

Hence heat flow into the base becomes :

$$Q = 2 \pi r_1 b k \sqrt{\beta} \theta_1 \frac{I_1(r_0 \sqrt{\beta}) K_1(r_1 \sqrt{\beta}) - K_1(r_0 \sqrt{\beta}) I_1(r_1 \sqrt{\beta})}{I_1(r_0 \sqrt{\beta}) K_0(r_1 \sqrt{\beta}) + K_1(r_0 \sqrt{\beta}) I_0(r_1 \sqrt{\beta})} \quad (2-9)$$

$$\text{where } \beta = 2h/(kb) \quad (2-10)$$

Gardner [6] has generalised the above relationship for extended surfaces of many configurations. These results are useful in the design of extended surfaces and presented in the

form of graph in terms of the effectiveness based on a similar fin of infinite conductivity.

#### 2.4 HEAT TRANSFER COEFFICIENT

The heat transfer coefficient for all surfaces is assumed to be the same. It has been observed that fin spacing affects the rate of heat transfer considerably. For any finned tube, the rate of heat transfer generally increases with reduced inter-fin distance [17]. However, the volume of fins cannot be increased, as there is a limit on inter-fin space; the fins should be spaced in such a manner that adjoining fins do not interfere with each other.

Katz et al [15] used a modified Nusselt's relationship to predict the heat transfer coefficient for finned tubes with Freon-12 condensing around it. The results were found to be in agreement with the experimental values. They considered the finned tube as horizontal with vertical surfaces attached to it, for natural convection.

Assuming laminar flow with forced convection, the Nusselt's relationship for vertical plate can be used to predict the heat transfer coefficient for Reynolds number ranging from 5,000 to 20,000.

The Nusselt relation for heat transfer coefficient is:

$$h = 0.664 \ k_a (\text{Pr})^{1/3} \ \sqrt{U_a / (L)} \quad (2-11)$$

The characteristic length  $L$  can be defined for circular fins to be equal to the area of one surface of the fin divided by the height of the fin. Thus we get,

$$L = \frac{\pi (r_o^2 - r_1^2)}{4 \cdot (r_o - r_1)} \quad (2-12)$$

For small diameter tubes, as used in the condenser, if  $(r_o - r_1)$  can be approximated as equal to  $r_o$  ( $r_1$  being small), Eq. (2-12) gives the characteristic length as defined by Katz et al [15].

The values of heat transfer coefficient obtained theoretically using Eq. (2-12) were found to be in agreement with experiments for cross-flow [14].

## 2.5 FCFV CONDITIONS

The Reynolds number of flow is defined as :

$$Re = \frac{UL}{\nu} \quad (2-13)$$

The characteristic dimension  $L$  here is defined as :

$$L = \frac{(d_o + d_1)}{2.0} \quad (2-14)$$

For a solid cylinder placed in a stream of air, both the outer and inner fin diameters become equal and we obtain the cylinder diameter as the characteristic length,

$$L = d_o \quad (2-15)$$

## CHAPTER III

## FORMULATION OF THE 'OPTIMIZATION PROBLEM'

3.1 INTRODUCTION

Methods of variational calculus can be used to find the optimum point  $\tilde{x}$  of a function  $f(\tilde{x})$ , when there are simple restrictions on the choice of  $\tilde{x}$ . But in practice, many restrictions are superimposed on the choice of  $\tilde{x}$ . They limit the range of permissible values of  $\tilde{x}$ , make the process of finding  $\tilde{x}$  complicated. Many methods have been developed to tackle such situations depending primarily on the nature of the function and the restrictions.

The restrictions imposed on the choice of the design vector  $\tilde{x}$ , are called constraints. The constraints  $g_j(\tilde{x})$ , may be of the form of inequality or equality, such that,

$$g_j(\tilde{x}) \leq 0, \quad (3-1)$$

$$j = 1, \dots, n$$

The domain obtained by the hyper surfaces defined by the constraints divides region into two, one of which satisfies all the constraints defined by Eq. (3-1), is called feasible region and every point in this region is the feasible point. The optimum point is a feasible point where the function value is a minimum.

The direct methods, such as the method of feasible directions, and indirect methods are used for non-linear functions.

Methods of feasible directions consist of finding a usable feasible direction and optimum step size for each step of the iteration. The techniques of finding a usable feasible direction is quite complicated and time consuming. The gradient projection method is effective for linear constraints.

Linear programming approach is very useful for solving problems where constraints as well as objective function are linear.

The indirect methods are useful where function and constraints are complicated. It includes the use of the interior or the exterior penalty functions.

The heat transfer constraint, which uses the Bessel functions, makes the problem complicated. The interior penalty function is preferred to others, when the initial point can be chosen in the feasible region. The subsequent points obtained during minimization will then necessarily lie in the feasible domain, owing to the nature of the penalty function. The unconstrained minima, finally converge to the relative minimum.

### 3.2 MOTIVATION FOR PENALTY FUNCTION

Consider some constraints on  $\mathbf{x}$  of function  $F(\mathbf{x})$ . Then, the problem becomes :

Minimize  $F(\underline{x})$ ,

subjected to

$$g_j(\underline{x}) \leq 0, \quad (3-2)$$

$j = 1, \dots, n.$

The minima of the function could be ensured by solving a set of differential equations :

$$\frac{d \underline{x}(\alpha)}{d \alpha} = - \nabla F(\underline{x}) \quad (3-3)$$

The function will approach the minimum point, where

$$\nabla F = 0 \quad (3-4)$$

The effect of constraints is yet to be incorporated in Eq. (3-3) and the approach adopted is to add something to Eq. (3-3) which would bend the trajectory to the feasible region.

Hence

$$\frac{d \underline{x}}{d \alpha} = - \nabla F(\underline{x}) - \sum_j M_j \nabla g_j(\underline{x}), \quad (3-5)$$

$$M_j = M_j(\underline{x}) = \begin{cases} 0 \\ r \end{cases} \quad \left. \begin{array}{l} g_j(\underline{x}) \leq 0, \\ g_j(\underline{x}) > 0. \end{array} \right\} \quad (3-6)$$

As  $M_j$  is discontinuous, the above set of equations may not necessarily give a solution, but this could be a platform to search for a penalty approach and constrained problems could be reduced to unconstrained ones as in Eq. (3-5).

The penalty function thus constructed is sequentially minimized until the optimum is reached, by varying the penalty parameter.

We define the penalty function

$$P(\tilde{\mathbf{x}}, r_k) = F(\tilde{\mathbf{x}}) - r_k \sum_{j=1}^m 1.0/g_j(\tilde{\mathbf{x}}) \quad (3-8)$$

where  $r_k$  is a penalty parameter.

The minimization of Eq. (3-8) is sought for a decreasing sequence of penalty parameter  $r_k$ . The penalty function always has a minimum in the feasible region, hence the relative minimum is forced towards the constrained minimum from the interior region.

As long as the optimum point,  $\tilde{\mathbf{x}}_1$ , is in the feasible region, the penalty term will add a small quantity to the objective function  $F(\tilde{\mathbf{x}})$ , but as some boundary is approached, one of the  $g_j(\tilde{\mathbf{x}})$  shoots up and the search direction will be changed. If  $r_k$  is made smaller in each cycle of minimization then, at a point  $\tilde{\mathbf{x}}_L$  the magnitude of penalty term becomes insignificant compared with the objective function and the point  $\tilde{\mathbf{x}}_L$  is accepted as the minimum of  $F(\tilde{\mathbf{x}})$ .

The initial value of  $r_k$  is chosen, such that, at the initial point, both the objective function and the penalty terms in Eq. (3-8) have almost the same weightage. Subsequently,  $r_k$  is decreased in each cycle by some fraction [20].

It is observed, that it takes 4-7 cycles to reach minimum.

#### ALGORITHM

1. Choose a feasible starting point  $\tilde{x}_0$ , satisfying the constraints

$$g_j(\tilde{x}) \leq 0. \quad (3-9)$$

2. Compute initial  $r_k$  such that :

$$r_k = \text{ABS} \left[ F(\tilde{x}_0) / \sum_j 1.0 / g_j(\tilde{x}) \right] \quad (3-10)$$

3. Construct the penalty function and obtain the optimum point,  $\tilde{x}_L$ , for  $P(\tilde{x}, r_k)$ , using some good technique for the unconstrained minimization.

4. If all convergence criteria are satisfied and  $r_k$  has been changed 3-4 times, then terminate; otherwise, change  $r_k$ , such that,

$$r_k \leftarrow r_k c \quad (3-11)$$

where  $c < 1$ .

$$\tilde{x}_0 = \tilde{x}_L \quad (3-12)$$

and repeat from step 3.

A flow chart of above scheme appears in Appendix A.

#### 3.3 UNCONSTRAINED MINIMIZATION

Many iterative schemes have been developed for the unconstrained minimization. The Grid and Random methods may not be able to locate exact minima, besides, these methods may need

several function evaluations. The zeroth order methods, univariate and Powell's do not require the derivatives of function. Powell's method is quadratically convergent and for the quadratic functions it converges in a few steps, but for the complicated functions it requires several cycles of minimization. Methods which use the first derivatives of function, are the first order methods, such as, steepest descent, Fletcher-Reeves or conjugate gradient methods. The steepest descent method is very slow because the successive moves are perpendicular. Fletcher-Reeves method is efficient for the quadratic functions but it takes a considerable time for complicated functions. Newton's procedure and Davidon-Fletcher-Powell method are second order methods. Newton's method needs the second order derivatives of the function and also inversion of matrices. The method is found to be strong, but not efficient. Sometimes it may be difficult to evaluate the second derivatives of the function. Davidon-Fletcher-Powell procedure does not require the second derivatives of function or matrix inversion, even then it is called second order method, as at the optimum point the H-matrix converges to the second order derivatives matrix. Davidon-Fletcher-Powell method, popularly known as D-F-P is quadratically convergent.

## ALGORITHM

1. Assume an initial positive definite symmetric matrix  $\begin{bmatrix} H_0 \end{bmatrix}$ , say  $\begin{bmatrix} I \end{bmatrix}$ ; corresponding to the initial guess vector  $\begin{bmatrix} X_0 \end{bmatrix}$ , find the search direction  $\begin{bmatrix} S_0 \end{bmatrix}$  :

$$\begin{bmatrix} S_0 \end{bmatrix} = - \begin{bmatrix} H_0 \end{bmatrix} \nabla F \left( \begin{bmatrix} X_0 \end{bmatrix} \right) \quad (3-13)$$

2. Use a powerful one dimensional minimization method to compute the optimum step-size  $\alpha^*$ , to obtain the new design vector  $\begin{bmatrix} X_{i+1} \end{bmatrix}$

$$\begin{bmatrix} X_{i+1} \end{bmatrix} = \begin{bmatrix} X_i \end{bmatrix} + \begin{bmatrix} S_i \end{bmatrix} \alpha^*, \quad (3-14)$$

where  $\alpha^*$  minimizes  $F \left( \begin{bmatrix} X_i + \alpha S_i \end{bmatrix} \right)$  one dimensionally.

3. If the convergence criterion is satisfied, then terminate; otherwise compute a new H-matrix  $\begin{bmatrix} H_{i+1} \end{bmatrix}$ , as

$$\begin{bmatrix} H_{i+1} \end{bmatrix} = \begin{bmatrix} H_i \end{bmatrix} + \begin{bmatrix} M_i \end{bmatrix} + \begin{bmatrix} N_i \end{bmatrix} \quad (3-15)$$

where

$$\begin{bmatrix} M_i \end{bmatrix} = \alpha^* \begin{bmatrix} S_i & S_i^T \end{bmatrix} / \begin{bmatrix} S_i^T & Y \end{bmatrix}, \quad (3-16)$$

$$\begin{bmatrix} N_i \end{bmatrix} = - \begin{bmatrix} (H_i Y_i) (H_i Y_i)^T \end{bmatrix} / \begin{bmatrix} Y_i^T & H_i Y_i \end{bmatrix}, \text{ and} \quad (3-17)$$

$$\begin{bmatrix} Y_i \end{bmatrix} = \nabla F \left( \begin{bmatrix} X_{i+1} \end{bmatrix} \right) - \nabla F \left( \begin{bmatrix} X_i \end{bmatrix} \right) \quad (3-18)$$

4. Compute the new search direction  $\begin{bmatrix} S_{i+1} \end{bmatrix}$  :

$$\begin{bmatrix} S_{i+1} \end{bmatrix} = - \begin{bmatrix} H_{i+1} \end{bmatrix} \nabla F \left( \begin{bmatrix} X_{i+1} \end{bmatrix} \right) \quad (3-19)$$

and repeat from step 2.

The optimum step length,  $\alpha^*$ , should be computed accurately to ensure that the new H-matrix generated is positive definite, for which the method will work efficiently and ensures  $F_{i+1} < F_i$ . The rounding off errors involved in computing  $\alpha^*$  accumulate and contaminate the search direction,  $s_{i+1}$ . Sometimes the process is restarted to overcome it [21]

The flow chart is given in Appendix A.

### 3.4 ONE DIMENSIONAL MINIMIZATION

One dimensional function,  $F(\alpha)$ , can be interpolated by a polynomial of any degree and the optimum of the polynomial can be computed. Quadratic interpolation method does not use the gradient of function, but it requires function evaluation at three points. Fibonacci search and Golden section methods, in general, require many function evaluations and are rarely used in practical problems. The direct root method uses the gradient and function evaluation at two points on either side of the minimum, and interpolate function linearly between these two points. Cubic interpolation method can be used if the first derivatives of the function are continuous. It uses the gradient and function evaluation at two points, such that,  $A \leq \alpha^* \leq B$ . Each refit will narrow the gap, (B-A), and ultimately the minimum will be reached. This method has been used here and the  $\alpha^*$  is computed

from the set :

$$Z = 3 \left\{ F(\tilde{x}_A) - F(\tilde{x}_B) \right\} / (B-A) + \nabla F(\tilde{x}_A) + \nabla F(\tilde{x}_B) \quad (3-20)$$

$$Q = \left\{ Z^2 - \nabla F(\tilde{x}_A) - \nabla F(\tilde{x}_B) \right\}^{1/2} \quad (3-21)$$

$$\alpha^* = A + (B-A) ( \nabla F(\tilde{x}_A) + Z + Q ) / ( \nabla F(\tilde{x}_A) + \nabla F(\tilde{x}_B) + 2Z ) \quad (3-22)$$

$$\text{and } \tilde{x}_{i+1} = \tilde{x}_i + \alpha^* (s_i) \quad (3-23)$$

The process of one dimensional minimization is repeated until convergence is obtained or the number of preassigned refits is completed.

The flow chart of one dimensional minimization is given in Appendix A.

### 3.5 CONVERGENCE CRITERIA

Several convergence criteria can be used to ensure the optimum point. The most suitable criteria are those, which are applied to the objective function and the design <sup>variables</sup> vector. These ensure that two consecutive minima do not differ much. These convergence criteria can be stated as :

$$\left| \frac{(\tilde{x}_{i+1} - \tilde{x}_i)}{\tilde{x}_{i+1}} \right| \leq E_1, \quad (3-24)$$

$$\text{and } \left| \left[ \frac{F(\tilde{x}_{i+1}) - F(\tilde{x}_i)}{F(\tilde{x}_{i+1})} \right] \right| \leq E_2 \quad (3-25)$$

One of the best criteria used for the termination of D-F-P method is

$$\left| \begin{bmatrix} \nabla F(\tilde{x}_i) \end{bmatrix}^T \begin{bmatrix} H_{i-1} \end{bmatrix} \begin{bmatrix} \nabla F(\tilde{x}_i) \end{bmatrix} / F(\tilde{x}_i) \right| \leq E_3 \quad (3-26)$$

The gradient at the minimum should be zero and it serves as one of the termination criteria for one dimensional cubic interpolation.

$$\left| \begin{bmatrix} s_{i+1} \end{bmatrix}^T \begin{bmatrix} \nabla F(\tilde{x}_{i+1}) \end{bmatrix} \right| \leq E_4 \quad (3-27)$$

To ensure accuracy of step-size,  $\alpha^*$ , one more termination criterion the orthogonality test, is used in one-dimensional minimization

$$\left| \begin{bmatrix} s_i \end{bmatrix}^T \begin{bmatrix} \nabla F(\tilde{x}_{i+1}) \end{bmatrix} / \left\{ |s_i| |\nabla F(\tilde{x}_{i+1})| \right\} \right| \leq E_5 \quad (3-28)$$

The values of  $E_1, E_2, E_3, E_4, E_5$  have been chosen from 2 percent to 0.1 percent.

### 3.6 CONSTRAINTS

The various constraints enforced are on heat transfer, Reynolds number of flow, Biot number, the number of fins etc.

#### 3.6-1 HEAT TRANSFER CONSTRAINT :

The rate of heat transfer from finned tube ought to be many times that of a bare tube. For imposing this constraint, the heat transfer from the finned tube is written as :

$$Q \gg K Q' \quad (3-29)$$

where  $Q'$  is the heat transfer from a bare tube under flow conditions identical to those of the finned tube. The value of  $\Xi$  which is a constant ( $> 1$ ) is usually very high between 4-40, hence Eq. (3-29) becomes

$$Q \geq 40 Q' \quad (3-30)$$

if larger value of  $\Xi$  is used.

### 3.6-2 FLOW CONDITION :

The condition of forced convection in laminar flow has to be maintained, otherwise the equations used for computing heat transfer will become invalid. The characteristic length for calculating Reynolds number of flow has been given in Eq. (2-18). To have established flow, the velocity of flow can be varied from 1.5 m/s to 5 m/s and Reynolds number should not exceed 7000.

Thus :

$$Re \leq 7000.0. \quad (3-31)$$

### 3.6-3 BIOT NUMBER CONSTRAINT :

The heat transfer area can be increased by increasing the tube size or by providing fins on it. While any increase in the volume of tube is expensive due to high cost of copper, the use of fins of some highly conducting material, such as aluminium is advantageous. The limit of the conditions for which fins are advantageous is that heat transfer should increase while its cost decreases. For this the Biot number, based on fin thickness

and outer fin radius, should not exceed unity. Further, the accuracy of fin calculations depends on how accurately the Biot number is computed.

For putting a limit, the Biot number has been obtained for the optimum fin geometry of free convection [8] and taken as a reference, as the Biot number in forced convection will be further less :

$$h b/k \leq .00067 \quad (3-32)$$

$$h r_o/k \leq 0.03 \quad (3-33)$$

### 3.6-4 BOUNDARY LAYER INTERFERENCE :

To avoid boundary layer interference on the adjoining fins, the inter-fin distance should be slightly smaller than twice the boundary layer thickness, but should be appreciably large. Comparison between theory and experiment shows that as a good agreement between the two, it should be slightly smaller than  $d_1/3.0$ , together with Eq. (3-32), the minimum number of fins should be :

for a tube length of 30.0 cm,  

$$n = 30.0 / \{ 0.4 + .00067(k)/h \} \quad (3-34)$$
  

$$\approx 35$$

To specify a constraint

$$35.5 \leq n \leq 45.5 \quad (3-35)$$

The upper limit on the number of fins will check steep fall in inter-fin distance.

### 3.6-5 OBVIOUS CONSTRAINTS :

Apart from these, some obvious restrictions include non-negativity constraints on fin geometry and inter-fin distance :

$$d_o \geq d_1 \quad (3-36)$$

$$s \geq c \quad (3-37)$$

$$b \geq 0 \quad (3-38)$$

The constraints have been normalised. The actual partial derivatives for all the constraints and objective function have been computed. The finite difference method has been used for the heat transfer constraint since Bessel functions are involved in its calculations. Each function minimum in sequential optimization is checked for constraints, to make sure of its feasibility.

### 3.7 COMPUTER PROGRAMME

The computer programme listing has been given in Appendix A. The steps involved are described below :

1. At each value of design variable, the value of a function,  $f(X)$  specified below is evaluated :

$$f(X) = \pi (d_o^2 - d_1^2) b \cdot n/4.0 - r_k \sum_j 1.0/g_j(X) \quad (3-39)$$

where  $g_j(X)$  are constraints :

$$g_j(X) \leq 0 \quad (3-40)$$

2. The heat transfer coefficient is evaluated before function evaluation by using the Nusselt relationship, defined by Eq. (2-11).
3. The Reynolds number of flow is obtained, by using Eq. (2-13).
4. The Bessel functions have been computed from series, taking first 10 dominating terms of series.
5. For each value of  $r_k$ , the Davidon-Fletcher-Powell method is applied to find the unconstrained minimum. The process is repeated till the convergence criteria are satisfied.
6. The value of  $r_k$  is changed till <sup>the</sup> relative minimum is obtained, which satisfies vector termination criteria.

## CHAPTER IV

### ELECTRICAL NETWORK ANALOGY

#### 4.1 INTRODUCTION

The analytical solution, discussed in Chapter II for combined convective and radiative heat transfer from finned surface is quite difficult as it involves a complex mathematical formulation. Often, only the network method can be used, since it is quite simple and efficient. It is analysed here, under the same assumptions as listed in Section 2.2.

#### 4.2 HEAT TRANSFER FROM A FIN

Consider the fin shown in Fig. 4.1 and a differential ring at a radius  $r_B$ . On writing an energy balance across the ring, considering the fin as a cylinder, we obtain :

$$Q_{\text{cond.}} = 4 \pi k b (\theta_1 - \theta)_B / \log_e(r/r_1)_B - 4 \pi k b (\theta - \theta_2)_B / \log_e(r_2/r)_B \quad (4-1)$$

Convection from the upper and lower surfaces of the ring :

$$Q_{\text{conv.}} = 4 \pi r_B (r_2 - r_1)_B h_c (\theta - \theta_g)_B \quad (4-2)$$

As the radial thickness of the node is small, the convective heat transfer from both surfaces will be small compared with conduction into the node and hence the one-dimensional conduction heat transfer equation for cylinder can be used.

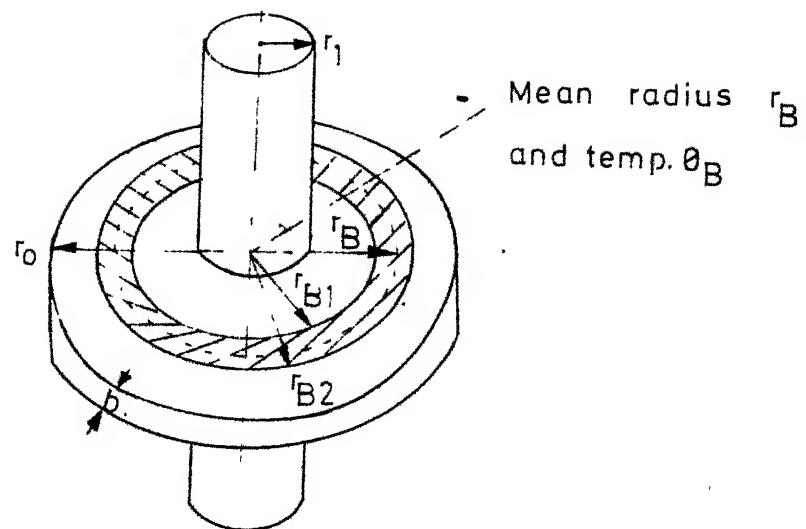


Fig. 4-1 Representation of node on the fin

Dent [11] obtained an agreement of 5-15% with those of experimental results using the one-dimensional heat conduction equation for cylinder.

The radiation from upper and lower surfaces of the ring to environment at temperature  $\theta_g$ , which is small, can be neglected.

Thus, energy balance on the ring :

$$\frac{(\theta_1 - \theta)_B}{\log_e(r/r_1)_B/(4\pi k b)} - \frac{(\theta - \theta_2)_B}{\log_e(r/r_1)_B/(4\pi k b)} = \frac{(\theta - \theta_g)_B}{1.0/4\pi h_c r_B (r_2/r_1)_B} \quad (4-3)$$

#### 4.3 NETWORK ANALOGY

The heat transfer equation in the preceding section can be equated to a network shown in Fig. 4.2. The application of Kirchhoff's law to this network yields :

$$\frac{V_{B1} - V_B}{R_{B1}} - \frac{V_B - V_{B2}}{R_{B2}} = \frac{V_B}{R_{II}} + \frac{V_B}{R_B} \quad (4-4)$$

One can see that Eq. (4-3) and Eq. (4-4) are analogous; comparison of both gives :

$$R_{B1} = \log_e(r/r_1)_B/(4\pi k b) \quad (4-5)$$

$$R_{B2} = \log_e(r_2/r)_B/(4\pi k b) \quad (4-6)$$

$$R_{II} = 1/(4\pi r_B (r_2 - r_1)_B h_c) \quad (4-7)$$

Hence it is seen that the fin can be represented by an equivalent network as shown in Fig. 4.3.

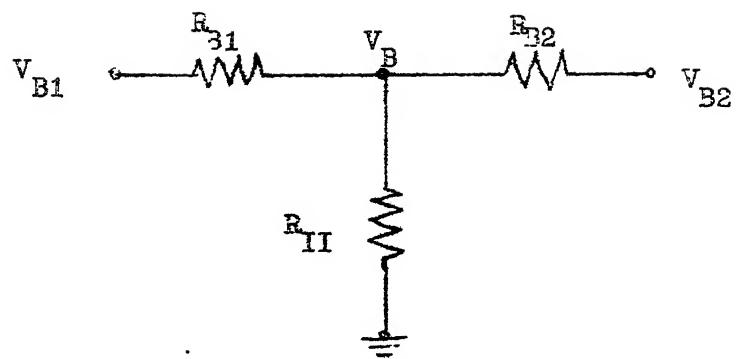


Fig. 4.2 : Network representation of a node

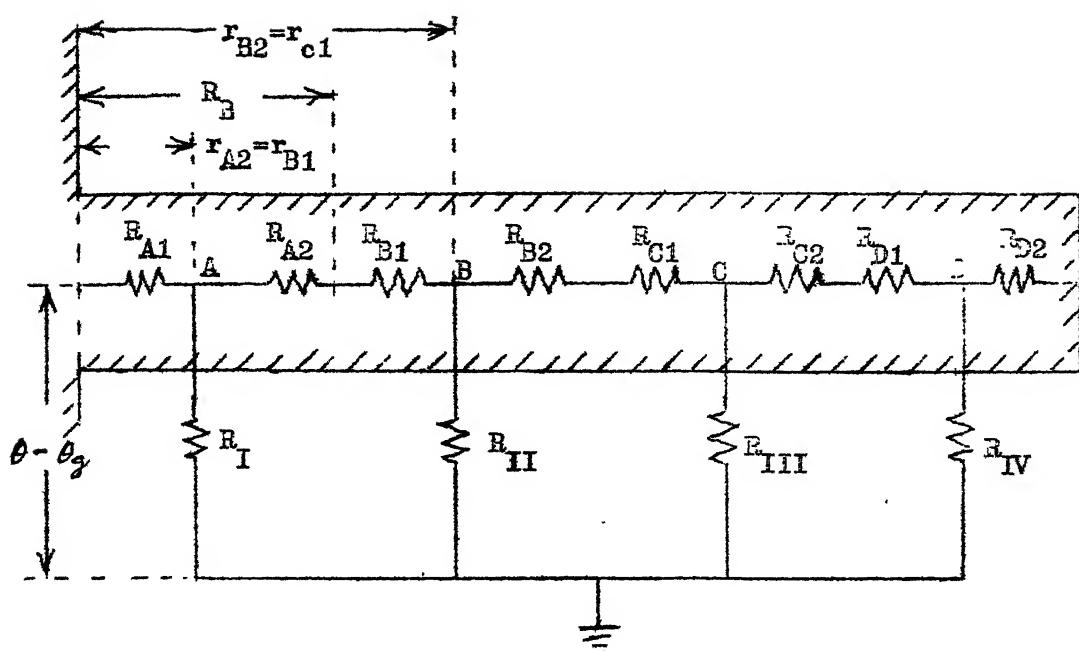


Fig. 4.3 : Network representation of the fin

#### 4.4 ITERATIVE SCHEME FOR SOLUTION OF NETWORK

To establish the temperature distribution in the fin, for an assumed value of the temperature distribution, heat transfer is computed by calculating the network resistances of the circuit. Once the total heat transfer (or current flow in circuit) is computed, actual temperature distribution through the network can be obtained. If it does not agree with the assumed value, the process is restarted with computed temperature distribution as the initial guess. The cycle is repeated until agreement is reached between the assumed and the computed temperature distribution. The convergence to the correct solution is very rapid.

When convergence is satisfied, the problem for total heat transferred by the fin has been solved and hence fin efficiency could be computed.

The number of nodes depends on the problem. As in other numerical techniques, the more the nodes used, the higher the accuracy but the more the labour involved in solution of the problem. Usually 5 to 8 nodes can be used as a compromise between accuracy and computer time involved.

A computer programme written for above iterative scheme appears in Appendix B.

## CHAPTER V

### RESULTS AND DISCUSSIONS

#### 5.1 INTRODUCTION

The optimum fin geometry and inter-fin distance have been obtained using optimization techniques for different temperature differences between the tube and surroundings. The results are presented in Table 1. Since the available wind tunnel for experimentation does not permit velocity of flow more than 4 m/s at the test section, and for laminar flow Reynolds number is to be maintained  $< 7000$ , all calculations have been carried out for a velocity of flow of 1.7 m/s.

The electrical network analogy has been used to obtain the radial temperature drop in the fin and verify the heat transfer rate.

#### 5.2 HEAT TRANSFER COEFFICIENT

The heat transfer coefficient was computed using Nusselt relationship for a vertical plate. The characteristic length has been defined in Eq. (2-12). Since the heat transfer rate from the tube is small (40 to 50 times) <sup>less</sup> in comparison to that of fins, the heat transfer coefficient for the tube was also assumed the same.

Figure 5-1 shows the variation of Nusselt number with  $1/(Re \cdot Pr)$  for Prandtl number of 0.72 as calculated by using

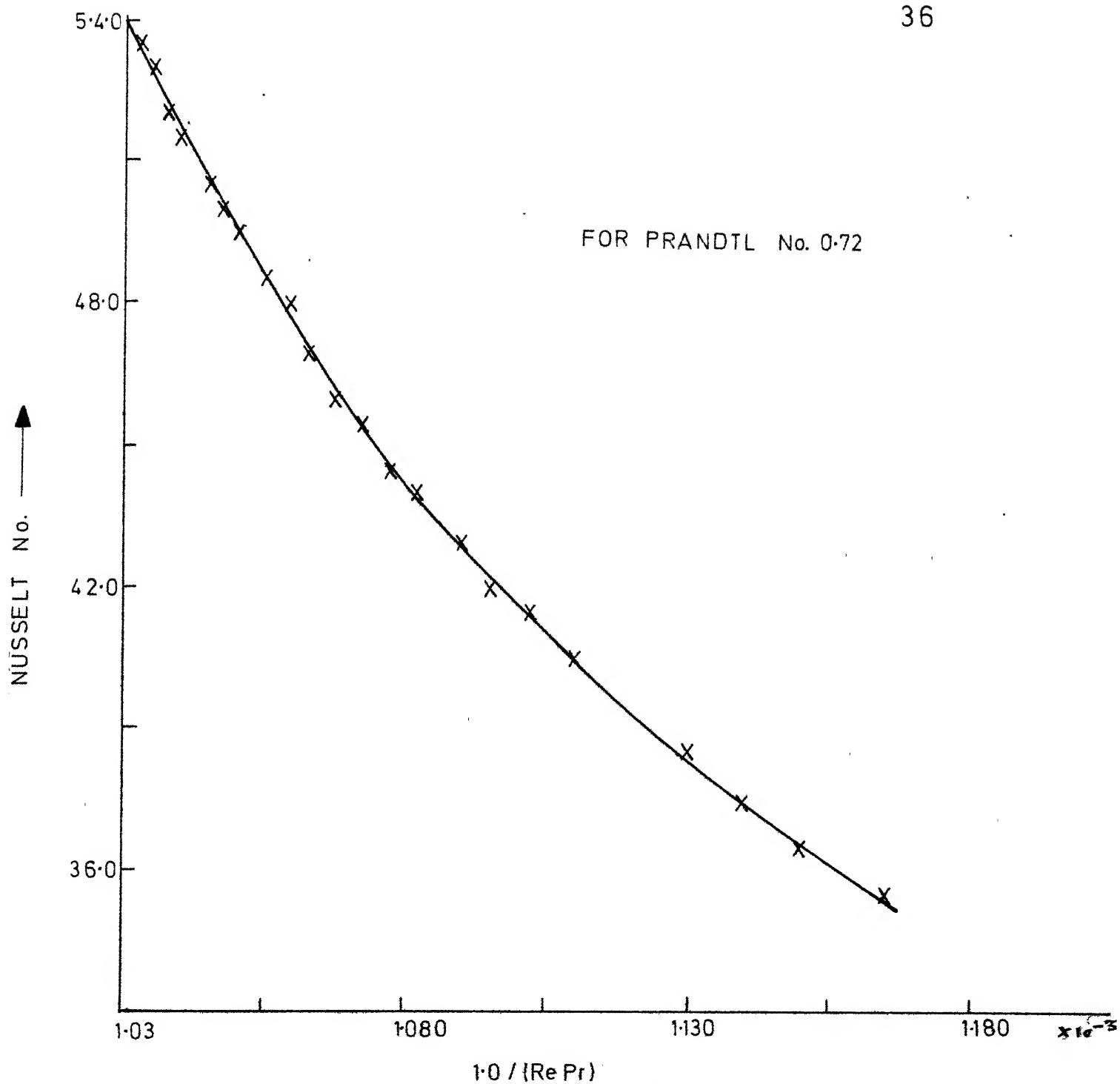


Fig. 5-1 Variation of Nusselt number with  $1.0 / (\text{Re Pr})$  for the fin

the Eq. (2-11) The results are found to be in agreement with those of constant wall temperature vertical plate [19]

Table 2 shows the effect of the inter-fin distance on the heat transfer coefficient. The heat transfer coefficient increases to some extent, if the inter-fin distance is decreased [2]. A rise of about 5% in the heat transfer coefficient is observed for a decrease of 67% in the inter-fin distance. If the inter-fin distance is less than  $d_1/12$  then, an increase in heat transfer coefficient is observed, even if inter-fin distance is increased. This establishes the effect of boundary layer interference on inter-fin distance and the heat transfer coefficient [2].

### 5.3 TEMPERATURE DISTRIBUTION IN THE FIN

Utcheon and Spalding [10] assumed the variation of the temperature as 5/4th power for the network method. The radial temperature distribution obtained by the numerical method and the network methods are presented in Table 3 and plotted in Fig 5-2.

An excellent agreement, within 2 percent, is found between the two methods.

### 5.4 OPTIMUM SOLUTION

Tables 1 and 2 contain the various values computed by the optimization programme. It is observed that the process converges to the minimum in 3 to 6 cycles and takes about 8 to 9 minutes, the compilation time of the programme being 4 minutes.

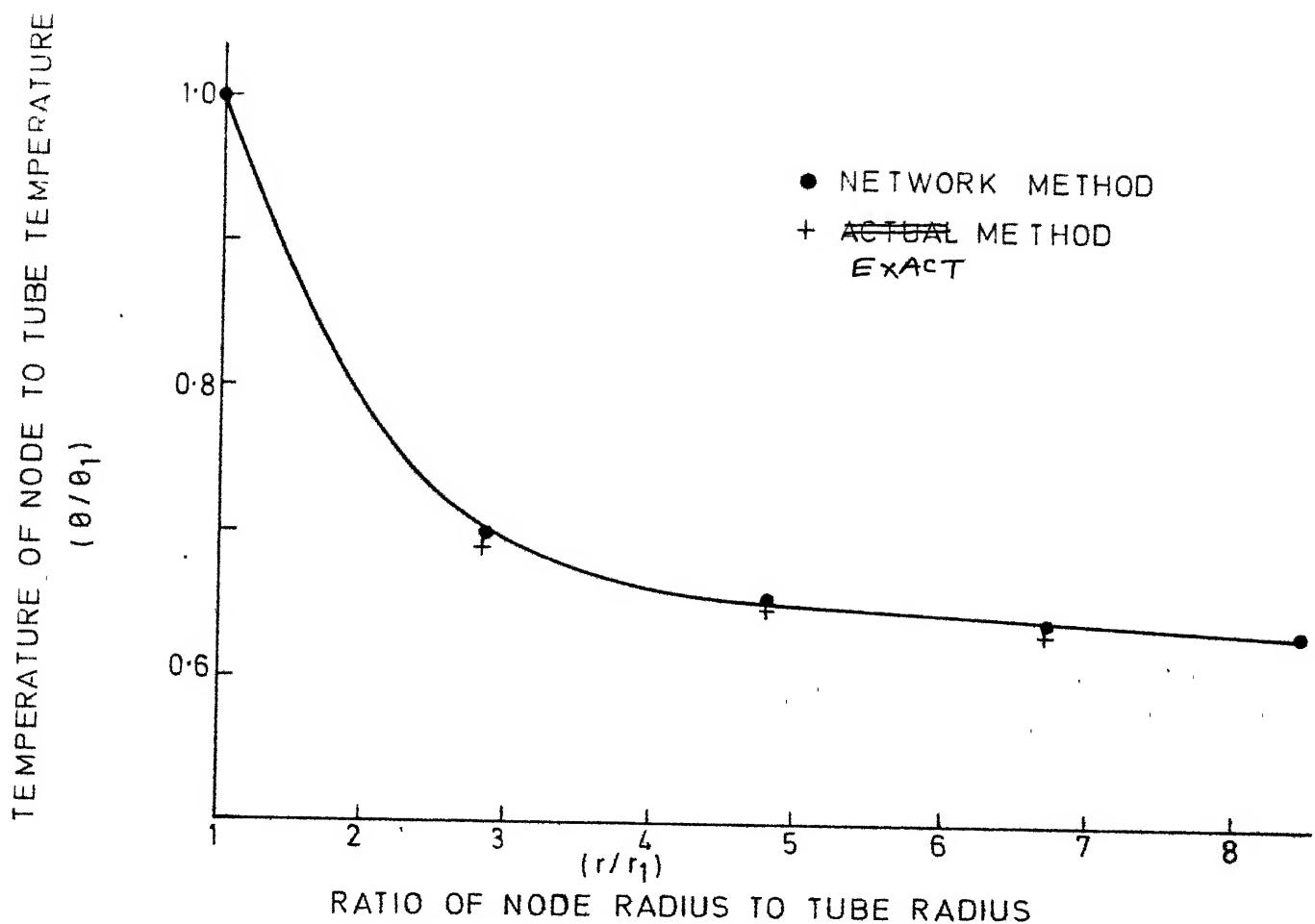


Fig. 5-2 Comparison of the radial temperature drop in the fin by the network and the actual methods

The execution time depends on the initial guess, which was the same for all temperature differences between the fins and surroundings.

The penalty function value has been reduced by 80-85 percent and the volume of the fins decreases by 40-50%. The heat transfer rate in comparison to that from bare tube under same condition of flow is 40-50 times more.

#### 5.5 HEAT TRANSFERRED FROM THE TIP OF THE FIN

The tip of the fin was assumed to be insulated for deriving boundary conditions for Eq. (2-6). Now accounting for the tip area of the fin; for the fin of maximum thickness :

$$\begin{aligned} \text{Ratio of tip area /fin area} &= \frac{\pi (10.88)(0.127)}{\pi (10.88^2 - 1.22^2)/2.0 + \pi (10.88)(0.127)} \\ &= 1.93\% \end{aligned} \quad (5-1)$$

which can be neglected in comparison to the total surface area.

#### 5.6 HEAT TRANSFERRED FROM THE FINNED TUBE

Figure. 5-3 shows the variation of the heat transfer with the temperature difference. The network method has been used to compute the heat transfer rate for the best fin. The deviation of 0.18% to 1.92% is observed in the heat transfer rate computed by two methods. The agreement is excellent, even when the equation for one dimensional heat transfer from a cylinder was used. This justifies use of the network method in comparison to the actual method which involves too much of computer time, while the network method is very efficient and fast.

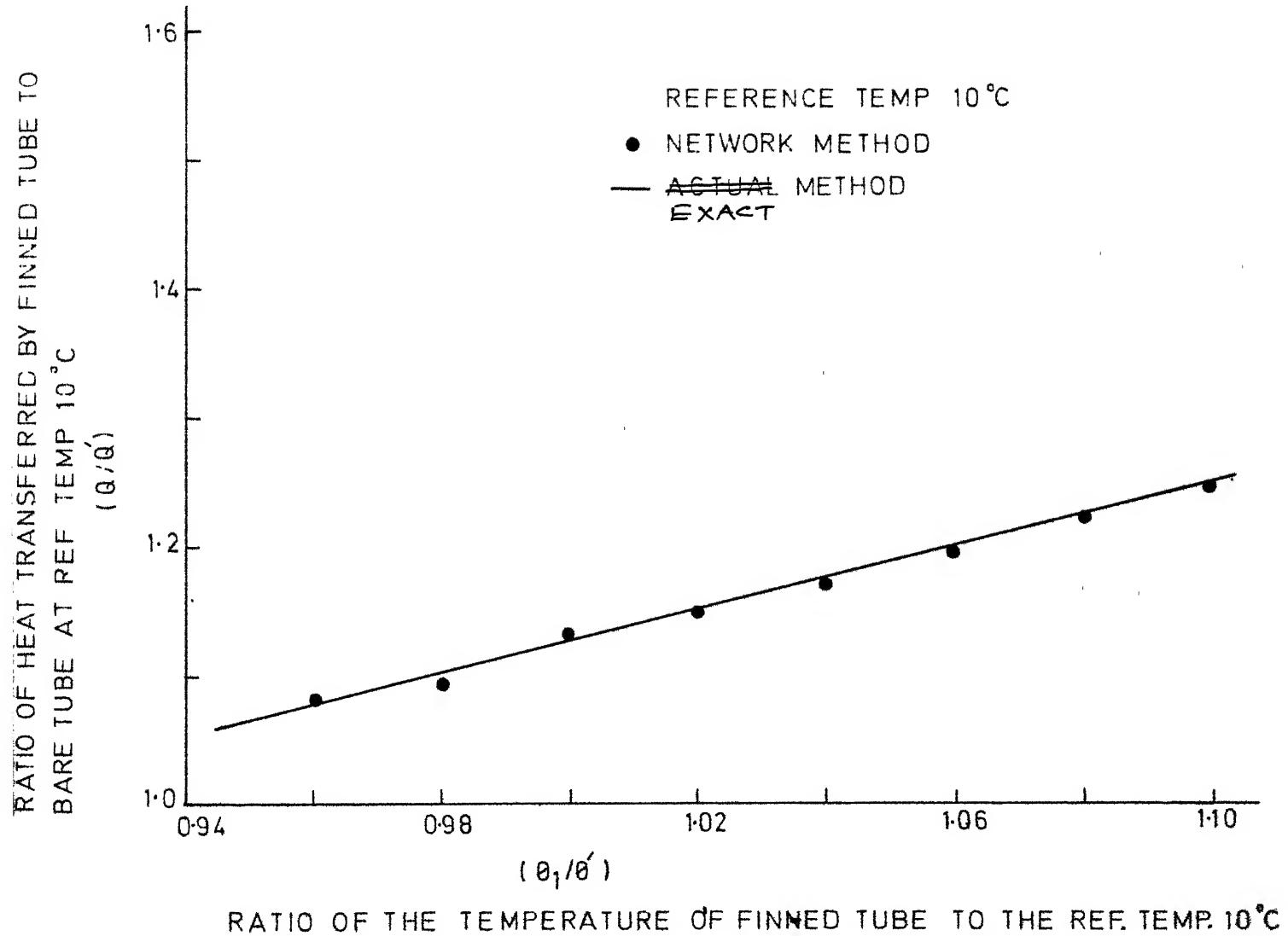


Fig. 5-3 Comparison of the heat transfer rate by the network and the actual methods

TABLE 1  
OPTIMUM FIN GEOMETRY AND INTER-FIN DISTANCE

REYNOLDS NUMBER	NO. OF FINS	OPTIMUM FIN GEOMETRY, CM			VOLUME OF FINS, CM <sup>3</sup>	FIN SPAC- ING, CM [cm <sup>2</sup> /cm]
		FIN DIA.	INTER-FIN DISTANCE	FIN THICKNESS		
3137	43	10.97	0.63	0.08	301.8	233.2
6750	43	10.27	0.583	0.127	402.2	200.5
6079	44	10.88	0.53	0.127	525.2	221.0

TABLE 2  
OPTIMIZATION RESULTS

Cycle Tem- pera- ture trained C minimi- zation		Penalty parameter	Penalty function	Volumetric of fins cm <sup>3</sup>	Heat trans- ferred kcal/hr	Inter- fin dis- tance cm	Heat trans- fer coeff. kcal/m <sup>2</sup> -C	Fin effi- citive ness	Com- puter time hrs.	
10	1	$1.89 \cdot 10^{-4}$	$7.1 \cdot 10^{-2}$	39	225.3	0.573	$1.285 \cdot 10^{-3}$	70.4	9	
	2	$9.5 \cdot 10^{-5}$	$4.6 \cdot 10^{-2}$		255.8	0.573	$1.291 \cdot 10^{-3}$			
	3	$4.74 \cdot 10^{-5}$	$2.6 \cdot 10^{-2}$		243.8	0.569	$1.30 \cdot 10^{-3}$			
	4	$2.0 \cdot 10^{-5}$	$2.2 \cdot 10^{-2}$		220.0	0.563	$1.33 \cdot 10^{-3}$			
	5	$8.5 \cdot 10^{-6}$	$1.39 \cdot 10^{-2}$		233.2	0.630	$1.345 \cdot 10^{-3}$			
3.3	1	$1.88 \cdot 10^{-4}$		742.0	41	229.0	0.568	$1.356 \cdot 10^{-3}$	70.2	9
	2	$9.42 \cdot 10^{-5}$		668.0	42	225.5	0.566	$1.37 \cdot 10^{-3}$		
	3	$4.71 \cdot 10^{-5}$		578.0	43	222.5	0.565	$1.38 \cdot 10^{-3}$		
	4	$3.06 \cdot 10^{-5}$		416.0	44	213.5	0.567	$1.385 \cdot 10^{-3}$		
	5	$1.99 \cdot 10^{-5}$		412.0	44	211.0	0.568	$1.383 \cdot 10^{-3}$		
	6	$1.29 \cdot 10^{-5}$	$1.85 \cdot 10^{-2}$	402.2	43	200.5	0.576	$1.39 \cdot 10^{-3}$		
7.2	1	$1.9 \cdot 10^{-4}$	$5.7 \cdot 10^{-2}$	40	212.0	0.60	$1.285 \cdot 10^{-3}$	70.0	9	
	2	$9.3 \cdot 10^{-5}$	$4.0 \cdot 10^{-2}$		211.8	0.573	$1.293 \cdot 10^{-3}$			
	3	$4.7 \cdot 10^{-5}$	$3.56 \cdot 10^{-2}$		220.4	0.564	$1.32 \cdot 10^{-2}$			
	4	$3.04 \cdot 10^{-5}$	$2.98 \cdot 10^{-2}$		221.0	0.56	$1.35 \cdot 10^{-3}$			

TABLE 3  
RADIAL TEMPERATURE DROP

TEMPERATURE DIFFERENCE BET B/W TUBE AND SURROUNDING		10 °C	
RADIUS OF TUBE		0.61 cm	
RADIUS OF FIN		5.49 cm	
NODE	$r/r_1$	$\theta/\theta_1$	
		Actual Method	Network Method
1	1	1	1
2	2.88	0.688	0.69
3	4.8	0.658	0.656
4	6.72	0.64	0.639
5	8.64	0.631	0.632

TABLE 4

COMPARISON OF THE HEAT TRANSFER RATE FOR  
OPTIMUM FIN GEOMETRY AT TEMPERATURE DIFFERENCE OF 10 C

Temp./Ref. $\theta/\theta_1$	Temp. $Q/Q_1$	Actual Method $Q/Q_1$	Resistance Method $Q/Q_1$	$\theta/\theta''$
0.02		1.041	1.039	1.131
0.04		1.064	1.061	1.131
0.06		1.088	1.083	1.132
0.08		1.110	1.105	1.132
1.00		1.132	1.135	1.132
1.02		1.155	1.151	1.132
1.04		1.179	1.172	1.132
1.06		1.20	1.198	1.132
1.08		1.223	1.22	1.132
1.10		1.245	1.242	1.132

## CHAPTER VI

### CONCLUSIONS AND SCOPE FOR FUTURE WORK

#### 6.1 CONCLUSION

An attempt has been made to apply the optimization techniques to the problems of heat transfer from fins. In the absence of the results for forced convection, no comparison with available information was possible. However, looking to the present practice of the use of thin fins placed close to each other in evaporator and heat exchanger, our results seem to be quite satisfactory. Even then an experimental verification is very important. Owing to the nature of the penalty function, the minimum obtained cannot be accepted as a relative minimum, but it always lies very close to relative minima.

The minimum obtained cannot be taken as an absolute minimum, to locate the absolute minimum, the initial guess is to be altered, to obtain several relative minima, if they exist.

Verifications by the network analogy method for the best fin has been found to be in agreement of between 0.2% to 2%, which is quite satisfactory. The network analogy method can be applied to compute the heat transfer rate and the temperature drop in the fin as the actual method requires much computer time.

The computer programmes are quite general and can be used for other problems with a little of modification.

#### 6.2 SCOPE FOR FUTURE WORK

The experimental verification on the available wind tunnel which can produce a flow of 1.5 m/s to 5 m/s can be taken as an

extension to this problem. Two 43 cm long tubes provided with the aluminium fins can be used. The radial temperature of the fin and tube can be measured by the thermocouples. The velocity of flow measured by an orifice meter is varied by an air blower. Electric or steam heating is available for experimentation.

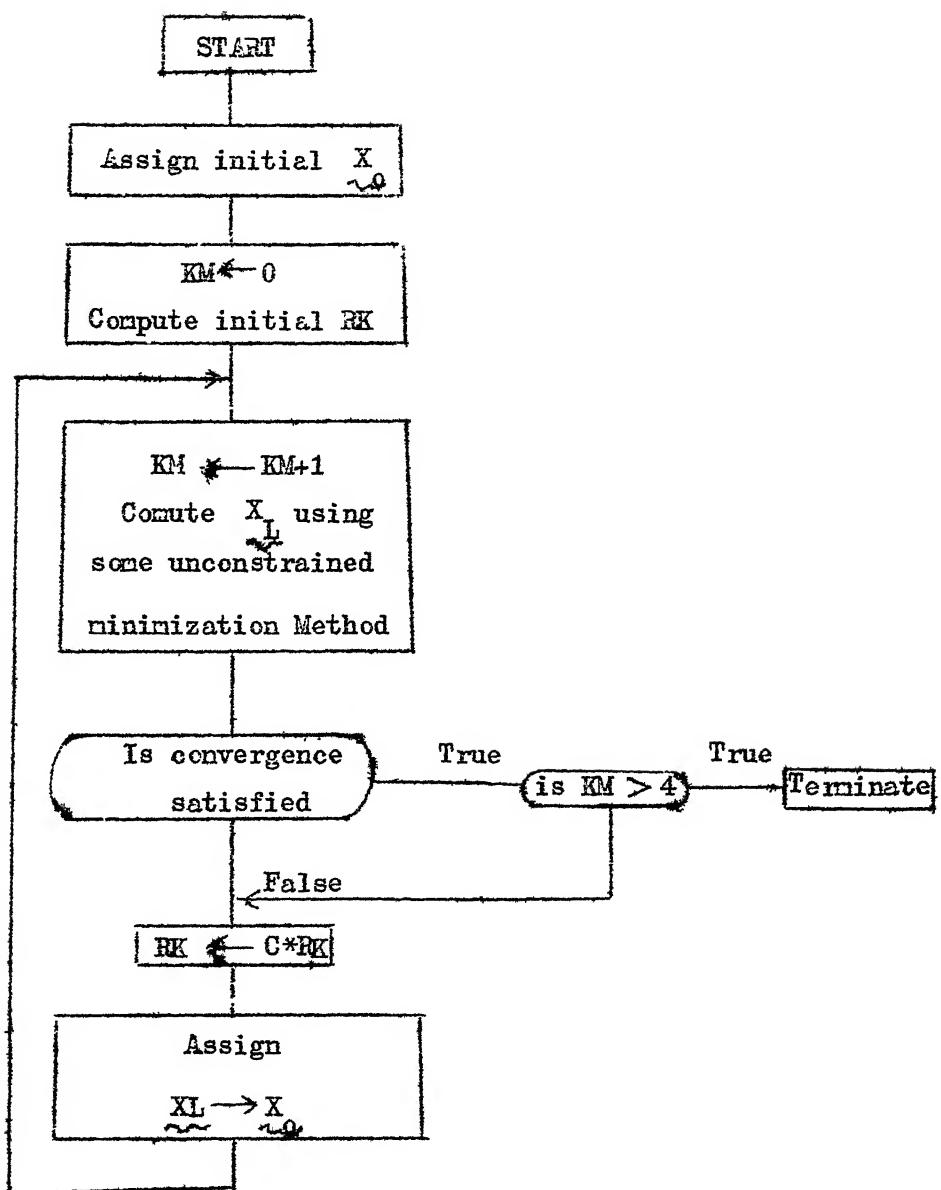
The effects of radiation have been neglected as the temperature difference was small. Optimization techniques may be used for high temperature differences by, accounting for the radiation. The shape factor can be obtained by solving the differential equation [5] with the help of some numerical technique.

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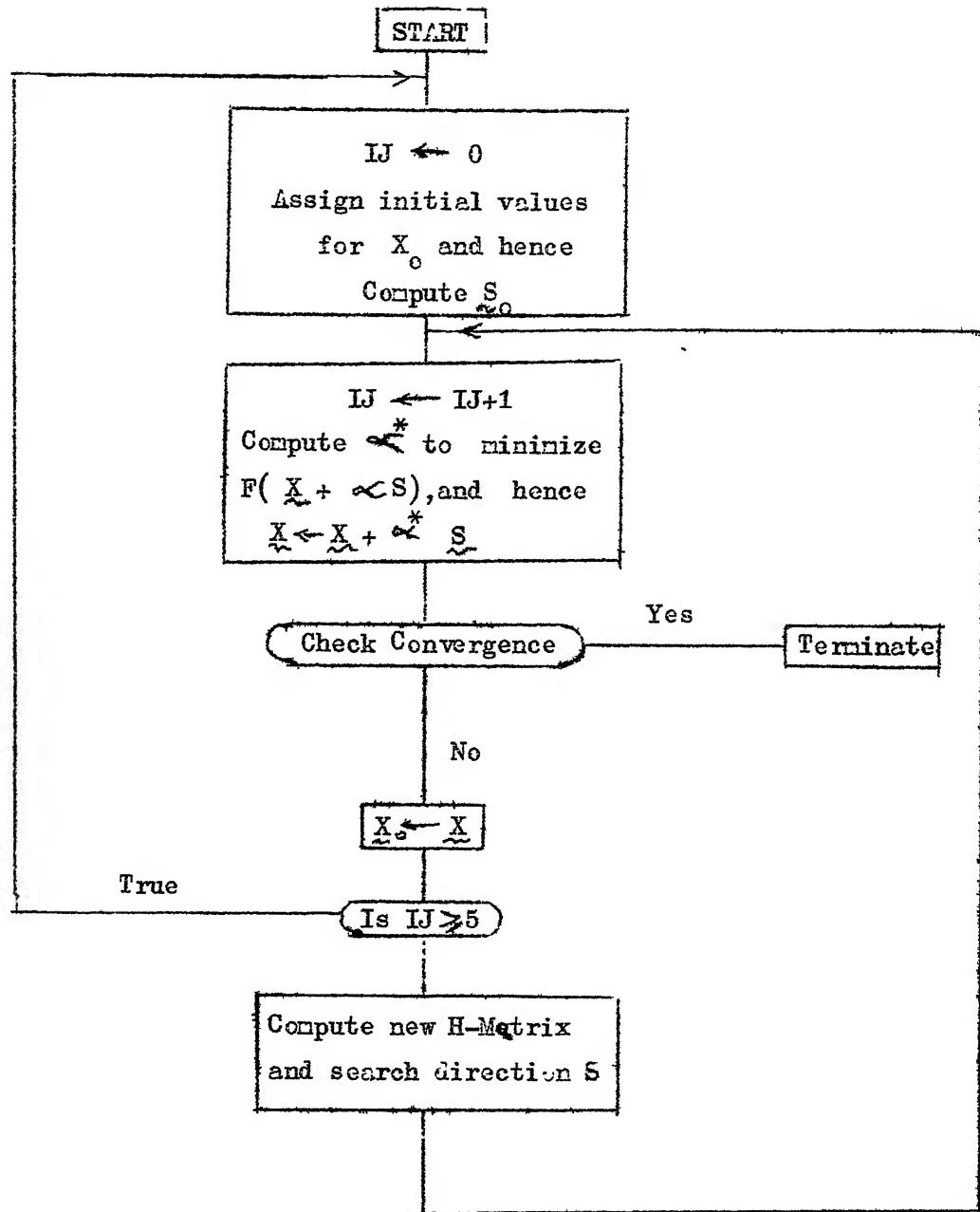
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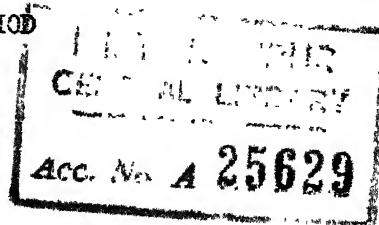
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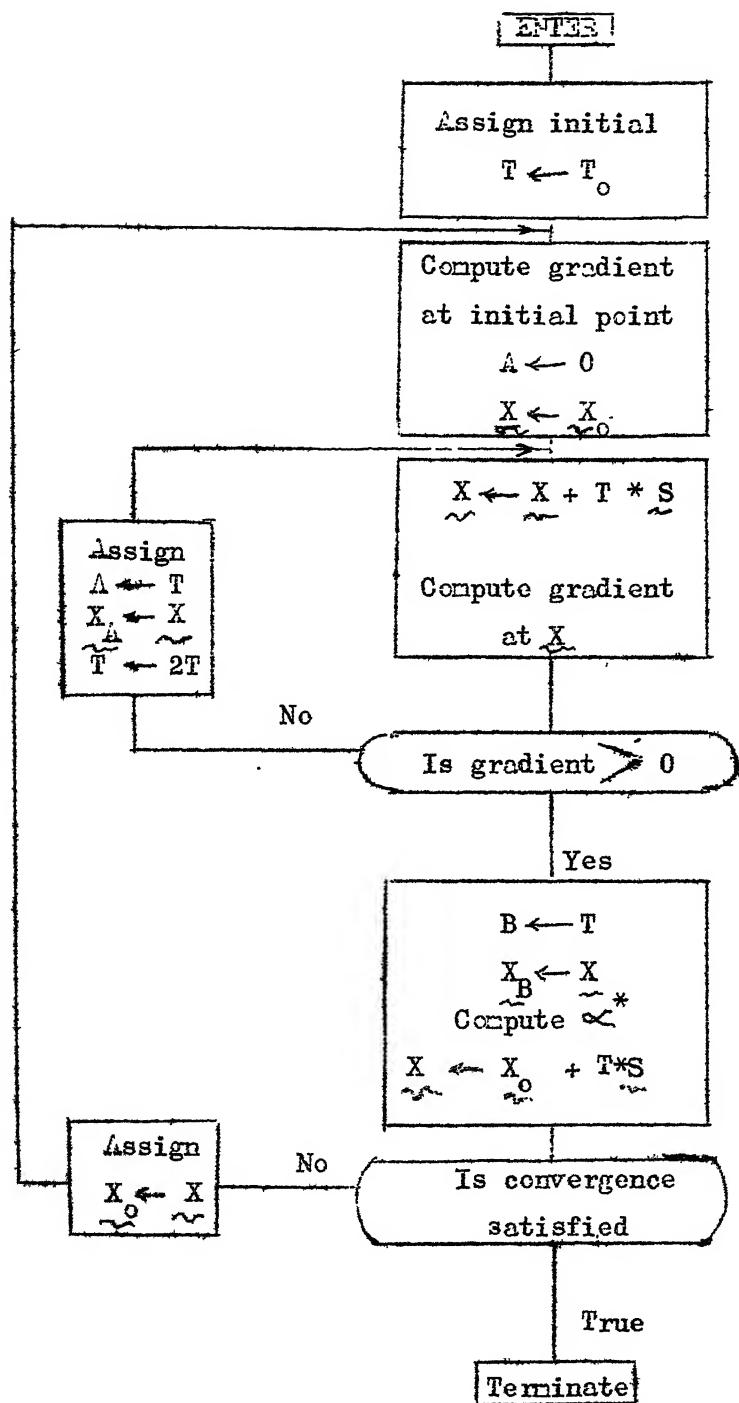
APPENDIX A

FLOW CHART FOR THE CONSTRAINED MINIMIZATION USING INTERIOR PENALTY METHOD



FLOW CHART FOR THE UNCONSTRAINED  
MINIMIZATION USING D-F-P METHOD





FLOW CHART FOR THE ONE-DIMENSIONAL MINIMIZATION  
USING CUBIC INTERPOLATION